Int J Theor Phys (2009) 48: 2550–2557
DOI 10.1007/s10773-009-0040-z

Domain Wall with Strange Quark Matter in Kaluza-Klein Type Cosmological Model

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Received: 12 March 2009 / Accepted: 8 May 2009 / Published online: 16 May 2009

Abstract In this study we have analyzed the Kaluza-Klein type Robertson Walker (RW) cosmological model by considering variable cosmological constant term Λ of the form: $\Lambda \sim \frac{\dot{R}^2}{R^2}$, $\Lambda \sim \frac{\ddot{R}}{R}$ and $\Lambda \sim \rho$ in the presence of strange quark matter with domain wall. The various physical aspects of the model are also discussed.

Keywords Strange quark matter · Bag constant · Higher dimensional space time

1 Introduction

A great number of exact cosmological solutions of Einstein field equations with different equation of state and different symmetries, including or not a cosmological constant, has been found with five dimensional (5D) [1–3] and also with arbitrary number of dimension [4–7]. Sahdev [8], Emelyanov et al. [9] and Chatterjee and Bhui [10], have studied physics of the universe in higher-dimensional space-time.

The possibility that the world may have more than the four dimensions is due to Kaluza [11] and Klein [12], who used one extra dimension to unify gravity and electromagnetism in a theory which was essentially five dimensional general relativity. This idea has been worked by a large number of people, who have found models for various phenomenon in particle physics and cosmology using 5D or more dimensions [13–16]. Overduin and Wesson [17] have presented an excellent review of Kaluza-Klein theory and higher-dimensional unified theories, in which the cosmological and astrophysical implications of

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extra-dimension have been discussed. Also, many authors have studied Kaluza-Klein cosmological models with different matters [18–23]. There is now extensive literature dealing with different aspect of higher dimensional cosmologist.

In this study, we will examine quark matter in the higher-dimensional space-time. It is well known that quark-gluon plasma existed during one of the phase transitions of the universe in the early time when the universe had higher dimensions than it has today and cosmic temperature was $T \sim 200$ MeV. Recently, quark matter and the relations between the quark matter and domain walls and also strings have been studied by several authors.

The possibility of the existence of quark-matter dates back to the early seventies. Itoh [24], Bodmer [25] and Witten [26] proposed two ways of formation of quark matter: the quark-hadron phase transition in the early universe and conversion of neutron stars into strange ones at ultrahigh densities. In the theories of strong interaction quark bag models, suppose that breaking of physical vacuum takes place inside hadrons. As a result, vacuum energy densities inside and outside a hadron become essentially different, and the vacuum pressure on the bag wall equilibrates the pressure of quarks, thus stabilizing the system.

Typically, strange quark matter is modeled with an equation of state (EOS) based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In this model, quarks are thought as degenerate Fermi gases, which exist only in a region of space endowed with a vacuum energy density B_c (called as the bag model). Also, in the framework of this model, the quark matter is composed of massless u, d quarks, massive s quarks and electrons. In the simplified version of this model, on which our study is based, quarks are massless and non-interacting. Then we have quark pressure $p_q = \frac{\rho_q}{3}$ (ρ_q is the quark energy density), the total energy density $\rho_m = \rho_q + B_c$ and the total pressure $p_m = p_q - B_c$. One therefore gets equation of the state for strange quark matter [27]

$$p_m = \frac{1}{3}(\rho_m - 4B_c). \tag{1}$$

We shall also use the following equation of state for the perfect fluid (normal matter)

$$p_m = (\gamma - 1)\rho_m, \tag{2}$$

where $1 \leq \gamma \leq 2$ is a constant.

In this paper we study domain wall attached to the strange quark matter and the normal matter in the context of Kaluza Klein theory of gravitation.

The cosmological constant problem can be expressed as the discrepancies between negligible values Λ has for the present universe and the values 10^{50} times larger expected by the Glashow Salam Weinberg model or by Grand Unified Theory (GUT) where it should be 10^{107} times larger. Some of the discussions on cosmological problem and on cosmology with time varying cosmological constant are investigated by Ratra and Peebles [28], Dolgov [29–31] and Sahni and Staro Binsky [32]. One of the motivations for introducing Λ term is to reconcile the age parameter and the density parameter of the universe with recent observational data.

Vishwakarma [33] has studied the magnitude-redshift relation for the type Ia supernovae data and the angular size-redshift relation for the updated compact radio sources data Gurvits [34] by considering four variable Λ-models: $\Lambda \sim R^{-2}$, $\Lambda \sim H^{-2}$, $\Lambda \sim \rho$ and $\Lambda \sim t^{-2}$.

Yilmaz et al. [35] studied strange quark matter for Robertson Walker model in the context of general theory of relativity. Also Yilmaz and Yavuz [36] have obtained higher dimensional Robertson Walker cosmological models in the presence of quark-gluon plasma in general theory of relativity. His work motivate one to consider further work in some alternatives theories of gravitation. In this context, the aim of the present work is based on recent available observational information. In this paper the implication of cosmological model with cosmological term of two different forms: $\Lambda = \alpha(\frac{\dot{R}}{R})^2$ and $\Lambda = \beta\rho$ are analyzed with strange quark matter within the framework of higher dimensional space time.

The paper is organized as follows: In Sect. 2, we have obtained the field equations for Kaluza-Klein type cosmological model in the presence of strange quark matter coupled to the domain wall. We have solved the same field equations for normal matter and strange quark matter by assuming cosmological constant Λ in the form of $\Lambda \sim \frac{\dot{R}^2}{R^2}$, $\Lambda \sim \frac{\ddot{R}}{R}$ and $\Lambda \sim \rho$ in Sect. 3. In Sect. 4, concluding remarks are given.

2 Model and Field Equations

Let us consider the Kaluza-Klein type metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2\right], \tag{3}$$

where $R(t)$ is the scale factor and $k = 0, -1$ or $+1$ is the curvature parameter for flat, open and closed universe, respectively. The universe is assumed to be filled with distribution of matter represented by energy-momentum tensor of a domain wall is given by

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - pg_{\mu\nu}. \tag{4}$$

The energy momentum tensor of the domain wall includes normal matter (describes by $\rho_m = \rho_q + B_c$ and $p_m = p_q - B_c$) as well as domain wall tension σ, i.e. $\rho = \rho_m + \sigma$ and $p = p_m - \sigma$. Also p_m and ρ_m are related by the bag model equation of state i.e. (1) and equation of state, i.e. (2). Here the five velocity vector such that $u_\mu u^\mu = 1$.

The Einstein field equations with time-dependent cosmological term Λ is given by

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} + \Lambda(t)g_{\mu\nu}, \tag{5}$$

where $R_{\mu\nu}$ is the Ricci tensor, $\Lambda(t)$ is the variable cosmological constants and G be the gravitational constant.

The divergence of (5), taking into account the Bianchi identity, gives

$$(8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu})^{;\nu} = 0. \tag{6}$$

Using co-moving coordinates

$$u_\mu = (1, 0, 0, 0, 0), \tag{7}$$

in (4) and with line element (3), Einstein's field equation (5) yields

$$8\pi G\rho = \frac{6\dot{R}^2}{R^2} + \frac{6k}{R^2} - \Lambda(t), \tag{8}$$

$$8\pi G p = -\frac{3\ddot{R}}{R} - \frac{3\dot{R}^2}{R^2} - \frac{3k}{R^2} + \Lambda(t), \tag{9}$$

where dot denotes derivative with respective to t and we will use the geometrized unit so that $8\pi G = c = 1$.

3 The Solutions of the Field Equations

In the above field equations the number of the unknowns is more than the equations, to be able to obtain an exact solutions of the field equations we assume the equations of the state for normal matter and strange quark matter with the cosmological constant Λ in the form of $\Lambda = \alpha H^2$, $\Lambda = \beta\rho$ and $\Lambda = \eta\frac{\ddot{R}}{R}$.

3.1 For $\Lambda = \alpha H^2$

The Einstein's field equations (8) and (9) with the above value of Λ can be expressed as

$$\rho = \rho_m + \sigma = \frac{(6-\alpha)\dot{R}^2}{R^2} + \frac{6k}{R^2}, \tag{10}$$

$$-p = -p_m + \sigma = \frac{3\ddot{R}}{R} + \frac{(3-\alpha)\dot{R}^2}{R^2} + \frac{3k}{R^2}. \tag{11}$$

From the above equations by eliminating ρ with the help of normal matter equations of the state described by (2), we get

$$R\ddot{R} + A_0\dot{R}^2 = -(2\gamma - 1)k. \tag{12}$$

After some mathematical steps we get the first integral

$$\dot{R}^2 = \frac{C_1}{R^{2A_0}} + \frac{A_1}{2A_0}, \tag{13}$$

where C_1 is the constant of integration and $A_0 = \frac{(6-\alpha)\gamma-3}{3}$ and $A_1 = -2k(2\gamma-1)$.

It is very difficult to find the exact solution of (13). However, by setting $C_1 = 0$, without loss of generality we are able to get the solution of (13). Hence integrating of (13) for the particular value of $\alpha = \frac{(\gamma-1)6}{\gamma}$ i.e. for $A_0 = 1$ we get

$$R^2 = \theta_0 + \theta_1(t-t_0)^2, \tag{14}$$

where t_0 is an constant of integration and $\theta_0 = \frac{C_1}{k(2\gamma-1)}$ and $\theta_1 = k(1-2\gamma)$.

From the above value of R, (10) and (11) with the help of Equation of state of normal matter (2), we get

$$\rho = \rho_m + \sigma = \frac{(6-\alpha)\theta_1^2(t-t_0)^2 + 6kX}{X^2}, \tag{15}$$

$$p = p_m - \sigma = (\gamma - 1)\left[\frac{(6-\alpha)\theta_1^2(t-t_0)^2 + 6kX}{X^2}\right], \tag{16}$$

where $X = \theta_0 + \theta_1(t-t_0)^2$. By adding (15) and (16) we get

$$\rho_m + p_m = \gamma\left[\frac{(6-\alpha)\theta_1^2(t-t_0)^2 + 6kX}{X^2}\right]. \tag{17}$$

By using Equation of state for quark matter, i.e. (1) we get

$$\rho_m = \frac{3\gamma}{4}\left[\frac{(6-\alpha)\theta_1^2(t-t_0)^2 + 6kX}{X^2}\right] + B_c, \tag{18}$$

or

$$\rho_q = \frac{3\gamma}{4}\left[\frac{(6-\alpha)\theta_1^2(t-t_0)^2 + 6kX}{X^2}\right], \tag{19}$$

$$p_q = \frac{\gamma}{4}\left[\frac{(6-\alpha)\theta_1^2(t-t_0)^2 + 6kX}{X^2}\right]. \tag{20}$$

Again with the help of (18) and (15)

$$\sigma = \left(1 - \frac{3\gamma}{4}\right)\left[\frac{(6-\alpha)\theta_1^2(t-t_0)^2 + 6kX}{X^2}\right] - B_c. \tag{21}$$

From the above equations we have the following expression for ρ_q and p_q and σ depending on the universe type ($k = 0, \pm 1$).

3.2 Case (i) $k = 0$

Substituting $k = 0$ in (12), we easily get the scale factor R. Then,

$$\rho_q = \frac{27}{4(6-\alpha)\gamma t^2}, \tag{22}$$

$$p_q = \frac{9}{4(6-\alpha)\gamma t^2}, \tag{23}$$

$$\sigma = \frac{9(4-3\gamma)}{4(6-\alpha)\gamma^2 t^2} - B_c. \tag{24}$$

3.3 Case (ii) $k = 1$

For $k = 1$ we get from (19)–(21)

$$\rho_q = \frac{3\gamma}{4}\left[\frac{(6-\alpha)\theta_{11}^2(t-t_0)^2 + 6X_1}{X_1^2}\right], \tag{25}$$

$$p_q = \frac{\gamma}{4}\left[\frac{(6-\alpha)\theta_{11}^2(t-t_0)^2 + 6X_1}{X_1^2}\right]. \tag{26}$$

Again with the help of (18) and (15)

$$\sigma = \left(1 - \frac{3\gamma}{4}\right)\left[\frac{(6-\alpha)\theta_{11}^2(t-t_0)^2 + 6X_1}{X_1^2}\right] - B_c, \tag{27}$$

where $X_1 = \theta_{01} + \theta_{11}(t-t_0)^2 = \frac{C_1}{(2\gamma-1)} + (1-2\gamma)(t-t_0)^2$.

3.4 Case (iii) $k = -1$

For $k = -1$ we get from (19)–(21)

$$\rho_q = \frac{3\gamma}{4}\left[\frac{(6-\alpha)\theta_{12}^2(t-t_0)^2 - 6X_2}{X_2^2}\right], \tag{28}$$

$$p_q = \frac{\gamma}{4}\left[\frac{(6-\alpha)\theta_{12}^2(t-t_0)^2 - 6X_2}{X_2^2}\right]. \tag{29}$$

Again with the help of (18) and (15)

$$\sigma = \left(1 - \frac{3\gamma}{4}\right)\left[\frac{(6-\alpha)\theta_{12}^2(t-t_0)^2 - 6X_2}{X_2^2}\right] - B_c, \tag{30}$$

where $X_{12} = \theta_{02} + \theta_{12}(t-t_0)^2 = \frac{-C_1}{(2\gamma-1)} + (2\gamma-1)(t-t_0)^2$.

3.5 For $\Lambda = \beta\rho$

In this case the field equations (8) and (9) can be expressed as

$$\rho(1+\beta) = \frac{6\dot{R}^2}{R^2} + \frac{6k}{R^2}, \tag{31}$$

$$-p + \beta\rho = \frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2}. \tag{32}$$

Again by using the equation state (2) for the normal matter the above field equations for the flat universe ($k = 0$) can be express as

$$\rho = \rho_m + \sigma = \frac{3(1+\beta)}{2\gamma^2 t^2}, \tag{33}$$

$$p = p_m - \sigma = \left[\frac{3(\gamma-1)(1+\beta)}{2\gamma^2 t^2}\right]. \tag{34}$$

Again by using equation of state (1) for the quark matter we get

$$\rho_m = \frac{9(1+\beta)}{8\gamma t^2} + B_c, \tag{35}$$

or

$$\rho_q = \frac{9(1+\beta)}{8\gamma t^2}, \tag{36}$$

$$p_q = \frac{3(1+\beta)}{8\gamma t^2}, \tag{37}$$

$$\sigma = \frac{3(1+\beta)(4-3\gamma)}{8\gamma^2 t^2} - B_c. \tag{38}$$

3.6 For $\Lambda = \eta\frac{\ddot{R}}{R}$

In this case the field equations (8) and (9) can be expressed with the help of (2) for flat universe ($k = 0$)

$$\rho = \rho_m + \sigma = \frac{3(\gamma\eta - 3)}{(\eta-6)\gamma^2 t^2}, \tag{39}$$

$$p = p_m - \sigma = \frac{3(\gamma-1)(\gamma\eta-3)}{(\eta-6)\gamma^2 t^2}. \tag{40}$$

Again by using equation of state (1) for the quark matter we get

$$\rho_m = \frac{9\gamma(\gamma\eta - 3)}{4(\eta - 6)\gamma^2 t^2} + B_c, \tag{41}$$

or

$$\rho_q = \frac{9\gamma(\gamma\eta - 3)}{4(\eta - 6)\gamma^2 t^2}, \tag{42}$$

$$p_q = \frac{3\gamma(\gamma\eta - 3)}{4(\eta - 6)\gamma^2 t^2}, \tag{43}$$

$$\sigma = \frac{3(\gamma\eta - 3)(4 - 3\gamma)}{4(\eta - 6)\gamma^2 t^2} - B_c. \tag{44}$$

4 Conclusion

We have obtained exact solutions of domain walls with quark matter by assuming variable cosmological form $\Lambda = \alpha H^2$, $\Lambda = \beta\rho$, $\Lambda = \eta\frac{\ddot{R}}{R}$ in Kaluza-Klein theory of gravitation.

In case of $\Lambda = \alpha H^2$, we obtained the solution for $k = 0$, 1 and -1. For $k = 0$, we find out the physical parameter p_q, ρ_q, ρ_m, p_m and tension σ and also, ρ_q and p_q tends to infinity as $t \to 0$ and σ tension is constant (a negative tension). Similarly for $t \to \infty$, p_q, $\rho_q \to 0$ but the domain wall tension σ in the form of bag negative constant i.e. we get negative tension with bag constant ($\sigma = -B_c$) for domain wall.

For the case $k = 1$ and -1 from (27) and (30), it is observed that, for the case $\gamma = 1$ (i.e. $p_m = 0$), we get the tension σ in terms of bag constant in the form of $\sigma = \frac{6c_1 - \alpha(t-t_0)^2}{4[c_1 - (t-t_0)^2]^2} - B_c$, $\sigma = -\frac{6c_1 - \alpha(t-t_0)^2}{4[c_1 - (t-t_0)^2]^2} - B_c$ respectively. Similarly in case of $\gamma = 2$ (i.e. stiff matter case) for $k = 1$ and -1 we get negative tension in terms of bag constant in the form of $\sigma = -\frac{2c_1 + 9(4-\alpha)(t-t_0)^2}{2[\frac{c_1}{3} - 3(t-t_0)^2]^2} - B_c$, $\sigma = \frac{2c_1 + 9(4-\alpha)(t-t_0)^2}{2[\frac{c_1}{3} - 3(t-t_0)^2]^2} - B_c$ respectively.

Similarly for the case $\Lambda = \beta\rho$ and $\Lambda = \eta\frac{\ddot{R}}{R}$, for $\gamma = 2$ and $k = 0$, from (38) and (44), it is observed that negative tension with bag constant in the form of $\sigma = -\frac{3(1+\beta)}{16t^2}$, $\sigma = -\frac{3(2\eta-3)}{8(\eta-6)t^2}$ respectively. We can easily obtained the solution for $\Lambda = \beta\rho$ and $\Lambda = \eta\frac{\ddot{R}}{R}$ for the case $k = 1, -1$.

Finally we may conclude that the domain wall are invisible due to their negative masses i.e. negative tension.

Acknowledgements One of author (GSK) would like to thanks the Department of Mathematics, The University of Abant Izzet Baysal, Campus of Izzet Baysal, Golkoy Bolu 14285, Turkey, for providing warm hospitality and excellent facilities where this work is done. This work is supported by the Scientific and Technical Research Council of Turkey (TUBITAK).

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